**Extended Essay, Comment and Assessment Rubric - Mathematics**

**Criterion A: Focus and method**

This criterion focuses on the topic, the research question and the methodology. It assesses the explanation of the focus of the research (this includes the topic and the research question), how the research will be undertaken, and how the focus is maintained throughout the essay.

(Strands: Topic, Research question, Methodology)

In mathematics the title of the essay, which may be in the form of a question, a proposition or a statement, can by itself clearly describe the topic and/or aim of the essay. It must not be too long and any necessary clarification of it, together with a clear indication of the mathematical areas and the techniques, should be provided early in the essay.

For example, “Methods for approximating $\pi$ throughout history”. In this essay I will describe methods of approximating $\pi$ from the work of Archimedes to the use of infinite series, infinite products and continued fractions in subsequent periods.” In other words, the focus and purpose of the essay must be made clear to the reader and appropriately related to the knowledge and understanding in context. This is clearly demonstrated when the research question indicates the mathematical techniques to be applied.

The sources consulted must be sufficient and each must contribute to the research focus of the essay.

The essay must be set out in sequential form in the manner of good mathematical writing, that is each section following on from and connected to the previous one.

A sharply written clear focus and research question can help the student ensure the essay remains within 4,000 words.
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<td>The work does not reach a standard outlined by the descriptors below.</td>
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<tr>
<td>1–2</td>
<td>The topic is communicated unclearly and incompletely</td>
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<td>- Identification and explanation of the topic is limited; the purpose and focus of the research is unclear, or does not lend itself to a systematic investigation in the subject for which it is registered.</td>
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<td>- The research question is stated but not clearly expressed or too broad</td>
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<td>- The research question is too broad in scope to be treated effectively within the word limit and requirements of the task, or does not lend itself to a systematic investigation in the subject for which it is registered.</td>
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<td>- The intent of the research question is understood but has not been clearly expressed and/or the discussion of the essay is not focused on the research question.</td>
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<td>Methodology of the research is limited</td>
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<td>- The source(s) and/or method(s) to be used are limited in range given the topic and research question.</td>
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<td>- There is limited evidence that their selection was informed.</td>
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<td>3–4</td>
<td>The topic is communicated</td>
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<td>- Identification and explanation of the research topic is communicated; the purpose and focus of the research is adequately clear, but only partially appropriate.</td>
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<td>The research question is clearly stated but only partially focused</td>
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<td>- The research question is clear but the discussion in the essay is only partially focused and connected to the research question.</td>
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<td>Methodology of the research is mostly complete</td>
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<td>- Source(s) and/or method(s) to be used are generally relevant and appropriate given the topic and research question.</td>
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<td>- There is some evidence that their selection(s) was informed.</td>
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<td>If the topic or research question is deemed inappropriate for the subject in which the essay is registered no more than four marks can be awarded for this criterion.</td>
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<tr>
<td>5–6</td>
<td>The topic is communicated accurately and effectively</td>
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<td>- Identification and explanation of the research topic is effectively communicated; the purpose and focus of the research is clear and appropriate.</td>
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<td>The research question is clearly stated and focused</td>
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<td>- The research question is clear and addresses an issue of research that is appropriately connected to the discussion in the essay.</td>
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<td>Methodology of the research is complete</td>
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<td>- An appropriate range of relevant source(s) and/or method(s) have been applied in relation to the topic and research question.</td>
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<td>- There is evidence of effective and informed selection of sources and/or methods.</td>
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Criterion B: Knowledge and understanding

This criterion assesses the extent to which the research relates to the subject area/discipline used to explore the research question, or in the case of the world studies extended essay, the issue addressed and the two disciplinary perspectives applied, and additionally the way in which this knowledge and understanding is demonstrated through the use of appropriate terminology and concepts.

(Strands: Context, Subject-specific terminology and concepts)

The essay must show clear evidence of understanding of the mathematics that is relevant to the focus of the essay. Students will not be rewarded for attempting to exhibit a wider knowledge of mathematics that is not essential to exploring the research question.

For example, in an essay on fractals, students must describe the mathematical concepts that underlie them without resorting to advanced theorems and results in analysis.

Students can demonstrate their understanding by:
- giving accurate and complete explanations of subject-specific terminology
- making knowledgeable comments on source material
- using source material in a relevant and appropriate way.

Students should ensure that the essay’s content is accessible to readers with a strong interest in the subject as well as to those with an advanced knowledge of it.

Students need to clearly communicate and explain their mathematics. They must not just talk about it but actually do the mathematics, and must show all steps in mathematical reasoning to make it clear that they understand it.

Students must make sure definitions are fully explained. If a theorem is used whose proof is too difficult, it should at least be explained by a clear example. Throughout, students need to demonstrate that they fully understand what they are doing.
### Criterion B: Knowledge and understanding. *The Assessment Criteria*

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| 1–2   | Knowledge and understanding is limited.  
- The selection of source material has limited relevance and is only partially appropriate to the research question.  
- Knowledge of the topic/discipline(s)/issue is anecdotal, unstructured and mostly descriptive with sources not effectively being used.  
Use of terminology and concepts is unclear and limited.  
- Subject-specific terminology and/or concepts are either missing or inaccurate, demonstrating limited knowledge and understanding. |
| 3–4   | Knowledge and understanding is good.  
- The selection of source material is mostly relevant and appropriate to the research question.  
- Knowledge of the topic/discipline(s)/issue is clear; there is an understanding of the sources used but their application is only partially effective.  
Use of terminology and concepts is adequate.  
- The use of subject-specific terminology and concepts is mostly accurate, demonstrating an appropriate level of knowledge and understanding. |
| 5–6   | Knowledge and understanding is excellent.  
- The selection of source materials is clearly relevant and appropriate to the research question.  
- Knowledge of the topic/discipline(s)/issue is clear and coherent and sources are used effectively and with understanding.  
Use of terminology and concepts is good.  
- The use of subject-specific terminology and concepts is accurate and consistent, demonstrating effective knowledge and understanding. |

If the topic or research question is deemed inappropriate for the subject in which the essay is registered no more than four marks can be awarded for this criterion.
Criterion C: Critical thinking

This criterion assesses the extent to which critical-thinking skills have been used to analyse and evaluate the research undertaken.

(Strands: Research, Analysis and Discussion and evaluation)

Students should be aware of the particular demands of critical thinking in mathematics.

At each opportunity in the essay, students must demonstrate their abilities in:
- correct deductive reasoning and argument
- establishing hypotheses
- formulating mathematical models.

For example, in the use of statistics to establish a hypothesis, students must collect the correct data, then display summary data and graphs, so that they choose, apply and interpret correctly the appropriate test or tests.

Students’ discussion and evaluation of their results should be concise.

It is important that students do the mathematics rather than merely describe it. They must show the steps in the algebra to demonstrate that they really understand what is going on. If they take any element from a source, they must cite that source.

Students should prove conjectures that can readily be proved. The essay must not just quote results; there must be evidence of the student doing mathematics.
**Criterion C: Critical thinking. The Assessment Criteria**

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| 1–3    | The research is limited.  
|        | - The research presented is limited and its application is not clearly relevant to the RQ.  
|        | Analysis is limited.  
|        | - There is limited analysis.  
|        | - Where there are conclusions to individual points of analysis these are limited and not consistent with the evidence.  
|        | Discussion/evaluation is limited.  
|        | - An argument is outlined but this is limited, incomplete, descriptive or narrative in nature.  
|        | - The construction of an argument is unclear and/or incoherent in structure hindering understanding.  
|        | - Where there is a final conclusion, it is limited and not consistent with the arguments/evidence presented.  
|        | - There is an attempt to evaluate the research, but this is superficial.  
|        | If the topic or research question is deemed inappropriate for the subject in which the essay is registered no more than three marks can be awarded for this criterion. |
| 4–6    | The research is adequate.  
|        | - Some research presented is appropriate and its application is partially relevant to the Research question.  
|        | Analysis is adequate.  
|        | - There is analysis but this is only partially relevant to the research question; the inclusion of irrelevant research detracts from the quality of the argument.  
|        | - Any conclusions to individual points of analysis are only partially supported by the evidence.  
|        | Discussion/evaluation is adequate.  
|        | - An argument explains the research but the reasoning contains inconsistencies.  
|        | - The argument may lack clarity and coherence but this does not significantly hinder understanding.  
|        | - Where there is a final or summative conclusion, this is only partially consistent with the arguments/evidence presented.  
|        | - The research has been evaluated but not critically. |
| 7-9 | The research is good.  
|     |  
|     | - The majority of the research is appropriate and its application is clearly relevant to the research question.  
|     | Analysis is good.  
|     | - The research is analysed in a way that is clearly relevant to the research question; the inclusion of less relevant research rarely detracts from the quality of the overall analysis.  
|     | - Conclusions to individual points of analysis are supported by the evidence but there are some minor inconsistencies.  
|     | Discussion/evaluation is good.  
|     | - An effective reasoned argument is developed from the research, with a conclusion supported by the evidence presented.  
|     | - This reasoned argument is clearly structured and coherent and supported by a final or summative conclusion; minor inconsistencies may hinder the strength of the overall argument.  
|     | - The research has been evaluated, and this is partially critical.  |
| 10-12 | The research is excellent.  
|     | - The research is appropriate to the research question and its application is consistently relevant.  
|     | Analysis is excellent.  
|     | - The research is analysed effectively and clearly focused on the research question; the inclusion of less relevant research does not significantly detract from the quality of the overall analysis.  
|     | - Conclusions to individual points of analysis are effectively supported by the evidence.  
|     | Discussion/evaluation is excellent.  
|     | - An effective and focused reasoned argument is developed from the research with a conclusion reflective of the evidence presented.  
|     | - This reasoned argument is well structured and coherent; any minor inconsistencies do not hinder the strength of the overall argument or the final or summative conclusion.  
|     | - The research has been critically evaluated.  |
Criterion D: Presentation

This criterion assesses the extent to which the presentation follows the standard format expected for academic writing and the extent to which this aids effective communication.

(Strands: Structure, Layout)

This criterion relates to the extent to which the essay conforms to accepted academic standards in relation to how research papers should be presented. It also relates to how well these elements support the reading, understanding and evaluation of the essay.

Students must provide a section and subsection structure to their essays, with appropriate informative headings. Students should aim to demonstrate their mastery of appropriate concepts and an ability to present these in an effective way using mathematical means. Concise, elegant mathematics supported by graphs, diagrams and important proofs that do not interrupt the development of the essay are encouraged.

Use of charts, images and tables

Diagrams and pictures should be in the text, immediately close to an explanation of them. Small data tables can be included in the body of the essay but larger ones should appear as an appendix, with means, standard deviations, correlation coefficients etc given in the text. Students should include computer routines only if they are absolutely necessary for the understanding of the essay. These must always appear as an appendix.

Any material that is not original must be carefully acknowledged, with specific attention paid to the acknowledgment and referencing of quotes and ideas. This acknowledgment and referencing is applicable to audiovisual material, text, graphs and data published in print and electronic sources. If the referencing does not meet the minimum standard as indicated in the guide (name of author, date of publication, title of source and page numbers as applicable), and is not consistently applied, work will be considered as a case of possible academic misconduct.

A bibliography is essential and has to be presented in a standard format. Title page, table of contents, page numbers, etc must contribute to the quality of presentation.

Word count is rarely an important factor in a good mathematics EE. Since equations and formulas (indicating the student’s mathematical reasoning) are not included in the word count, a substantial essay can be produced that contains comparatively few words.

Concise, elegant mathematics supported by graphs, diagrams and important proofs that do not interrupt the development of the essay are encouraged. However, an essay that is excessive in length will be penalized, especially if this is because of unnecessary content. Students should be aware that examiners will not read beyond the 4,000-word limit, or assess any material presented past this.

There is no mandatory minimum length for an essay in mathematics, and credit will be given for organizing the content in an efficient and readable style, rather than for a page or word count. Mastery of appropriate concepts, and an ability to present these in an effective way using mathematical means, should be the aim. Students should use an appendix as appropriate (eg for large amounts of raw data or for computer routines). However, any mathematics that is essential to the understanding of the essay must appear in the main body of the essay.
**Criterion D: Presentation. The Assessment Criteria**

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| 1–2   | Presentation is acceptable.  
- The structure of the essay is generally appropriate in terms of the expected conventions for the topic, argument and subject in which the essay is registered.  
- Some layout considerations may be missing or applied incorrectly.  
- Weaknesses in the structure and/or layout do not significantly impact the reading, understanding or evaluation of the extended essay. |
| 3–4   | Presentation is good.  
- The structure of the essay clearly is appropriate in terms of the expected conventions for the topic, the argument and subject in which the essay is registered.  
- Layout considerations are present and applied correctly.  
- The structure and layout support the reading, understanding and evaluation of the extended essay. |
Criterion E: Engagement

This criterion assesses the student’s engagement with their research focus and the research process. It will be applied by the examiner at the end of the assessment of the essay, after considering the student’s Reflections on planning and progress form.

(Strands: Reflections on planning and progress)

This criterion assesses the student’s engagement with their research focus and the research process. It will be applied by the examiner at the end of the assessment of the essay, and is based solely on the candidate’s reflections as detailed on the RPPF, with the supervisory comments and extended essay itself as context.

Students are expected to provide reflections on the decision-making and planning process undertaken in completing the essay. Students must demonstrate how they arrived at a topic as well as the methods and approach used. This criterion assesses the extent to which a student has evidenced the rationale for decisions made throughout the planning process and the skills and understandings developed.

For example, students may reflect on:
- the approach and strategies they chose, and their relative success
- the Approaches to learning skills they have developed and their effect on the student as a learner
- how their conceptual understandings have developed or changed as a result of their research
- challenges they faced in their research and how they overcame these
- questions that emerged as a result of their research
- what they would do differently if they were to undertake the research again.

Effective reflection highlights the journey the student has engaged in through the EE process. Students must show evidence of critical and reflective thinking that goes beyond simply describing the procedures that have been followed.

The reflections must provide the examiner with an insight into student thinking, creativity and originality within the research process. The student voice must be clearly present and demonstrate the learning that has taken place.
Criterion E: Engagement. *The Assessment Criteria*

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| 1–2   | Engagement is limited.  
|       | - Reflections on decision-making and planning are mostly descriptive.  
|       | - These reflections communicate a limited degree of personal engagement with the research focus and/or research process. |
| 3–4   | Engagement is good.  
|       | - Reflections on decision-making and planning are analytical and include reference to conceptual understanding and skill development.  
|       | - These reflections communicate a moderate degree of personal engagement with the research focus and process of research, demonstrating some intellectual initiative. |
| 5–6   | Engagement is excellent.  
|       | - Reflections on decision-making and planning are evaluative and include reference to the student’s capacity to consider actions and ideas in response to setbacks experienced in the research process.  
|       | - These reflections communicate a high degree of intellectual and personal engagement with the research focus and process of research, demonstrating authenticity, intellectual initiative and/or creative approach in the student voice. |